

GENERAL RELATIVITY, SPRING 2014
 Homework Set 8: Due start of class Tuesday, May 5th
Reading: Hartle Chapters 20,21 and 22

1. Hartle, Chapter 21, Q3
2. Consider the semi-log p, q co-ordinate system

$$p = x \quad q = e^{by} \tag{1}$$

(a) Calculate the Cartesian components of the basis vectors \vec{e}_p, \vec{e}_q of the p, q co-ordinate system, and hence calculate the metric.

(b) Calculate the one non-zero Christoffel symbol

Consider the vector field \vec{A} which has components $A^x = 0, A^y = Cx$ (i.e. $\vec{A} = (0, Cx)$) in Cartesian coordinates, where C is a constant.

(c) Calculate the components of \vec{A} in the p, q co-ordinate system.

(d) Calculate the gradient of the vector field $\nabla_i A^j$ in Cartesian coordinates

(e) Calculate the gradient of the vector field $\nabla_i A^j$ in the p, q coordinates

(f) Transform the gradient of \vec{A} from p, q coordinates to Cartesian co-ordinates, demonstrating that you end up with the same answer as you obtained in part (d).

3. Consider the two-dimensional line element

$$ds^2 = a^2 du^2 + \sin^2(u) dw^2 \tag{2}$$

where a is a constant.

(a) Find all the non-zero Christoffel symbols.

(b) Find all components of the Gradient $\nabla_i A^j$ of a vector field $\vec{A} = (A^u, A^w)$.

(c) Write down the form of the D'Alembertian of a scalar field $\Phi, \nabla_i \nabla^i \Phi$.

(d) Find all non-zero components of the Riemann tensor R_{ijkl}

(e) Find all non-zero components of the Ricci tensor R_{ij}

(f) Find the Ricci scalar R .

(g) Is this a curved space?

4. A rope made of nylon of density ρ and with cross-section A lies along the x -axis under tension F . Write down the form of the energy-momentum tensor inside the rope. Show that requiring the energy density of the rope to be positive for all observers limits the allowed tension F . (Hint: Lorentz-boost the energy-momentum tensor along the x -axis.)